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Prediction of Elastic Properties of Short Fibers Reinforced Composites

التنبؤ بخواص المرونة للمواد المركبة والمسلحة بشعيرات قصيرة

Ebraheem Shady
Department of Textile Engineering, Faculty of Engineering, Mansoura University
Mansoura 35516, Egypt
shady@mans.edu.eg

الملخص:

هناك استخدامات صناعية عديدة للمواد المركبة البوليمرية المسلحة بشعيرات صناعية قصيرة، ومن المهم وجود نموذج رياضي قادر على التنبؤ بخواص المرونة للمواد المركبة بدلاً من تصنيعها وإجراء تجارب عليها لمعرفة هذه الخواص. وجود مثل هذا النموذج سوف يوفر الجهد والوقت اللازم لتصميم هذه المواد. إضافة أن هذا النموذج الرياضي سيسمح بدراسة تأثير بعض المتغيرات على خواص المواد المركبة. تهدف هذه الدراسة إلى تطبيق وبرمجة نموذج رياضي ودمجه مع أسلوب لتوزيع الشعيرات عشوائياً في البوليمر. النموذج الرياضي مستنتج في الأساس من النموذج المعدل لـ Mori-Tanaka. تم التحقق من صحة النموذج بمقارنة نتائجه بنتائج عملية منشورة في المراجع العلمية. أثبتت المقارنة توافق كبير بين النتائج العملية ونتائج النموذج المقدم فقط عند استخدام أسلوب التوزيع العشوائي للشعيرات ليماثل واقع توزيع الشعيرات. كما تم بعد تحقيق النموذج الرياضي عمل دراسة حالة لتوضيح تأثير تغيير نسبة الشعيرات وتغيير النسبة بين طول الشعيرة لقطرها على الخواص الناتجة للمادة المركبة. أظهرت النتائج أنه بزيادة النسبة الحجميه للشعيرات يزداد بشكل غير خطي كل من معامل المرونة ومعامل القص بينما يقل بشكل غير خطي نسبة بواسون. كما أظهرت النتائج أن تغيير نسبة طول الشعيره لقطرها ليس له تأثير ملموس على خواص المواد المركبة.

Abstract

Short fiber reinforced polymer composites (SFRC) are used widely in industrial applications. It is imperative to have analytical expression that is able to predict elastic properties of composite materials instead of manufacturing and testing the materials. Having such expression saves efforts and time when designing composite materials. In addition, the analytical expression will enable examining the effects of some parameters on the elastic properties of composite materials. In this work, an analytical model is adopted to predict elastic properties of short fiber reinforced polymer composite. The fibers are considered aligned and randomly oriented in the matrix. The model is based on the modified Mori-Tanaka approach along with a randomization procedure. The model is verified by comparing its results to experimental data from literature. The model results that consider the random orientation of fibers agree well with the experimental data. Thereafter, the model is used to perform a case study to elucidate the effect of fiber volume fraction and fiber's aspect ratio on the elastic properties of the composite material. The results show that increasing fiber volume fraction leads to nonlinear increase in both elastic and shear moduli and non linear decrease in Poisson's ratio. The study shows insignificant effect of changing fiber's aspect ratio on the elastic properties of composite materials.

Keywords: *Short fiber, Polymer composites, Random orientation, Elastic modulus, Aspect ratio.*

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1. Introduction

Industrial applications of short fiber reinforced composites (SFRC) are largely expanded due to their high strength and stiffness to weight ratio. In addition, ease of fabrication and low cost elected them for a large scale of products. They are used in automotive industry, aerospace and biomedical applications. Several factors are governing the elastic behavior of short fiber composites such as fiber volume fraction, fiber packing arrangement, fiber geometry, fiber orientation and fiber-matrix interface. Mostly, short fibers are dispersed randomly in the polymer matrix due to comparative easiness of production process. Due to the wide scale applications of the SFRC, it is so important to predict the elastic properties of composite materials. Researchers have been working to develop theoretical base methods to predict elastic properties of SFRC that are considerably accurate when compared to experimental data [1-4].

Several models were developed to predict mechanical properties of aligned short fiber reinforced composites. Most widely used models that give considerably accurate results are based on shear-lag analysis which originally proposed by Cox [1], Halpin and Kardos [2], Agarwal et al. [3], and Tucker and Liang [4]. Applying these models on randomly oriented short fibers to predict the composite properties is limited. However, some empirical relationships can be used in conjunction with these models to determine approximate properties of random short fiber composite [3, 5]. Christensen and Waals [6] developed a model to examine the composite behavior considering three-dimensional (3-D) random orientation and fiber-matrix interaction effects. They claimed that the model provides reasonable results at low fiber volume fractions. The comparison between the model results at low volume fractions to experimental data by Lee [7] showed that the prediction was within a range of 0-15% higher than the experimental data. Manera [8] presented approximate equations to predict the elastic properties of randomly oriented short fiber glass composite in 2-D. It was assumed that the fibers have high aspect ratio (fiber length/fiber diameter >300), the fibers are randomly distributed in 2-D and the composites are treated as laminates with an infinite layers oriented in all directions. The results of the approximate equation that predicts composite modulus was compared to test data showing differences less than 5%. Pan [9] developed an approach to predict elastic properties based on the rule of mixture approach. Pan introduced a probability density function to specify the fiber orientation along with a constructed relationship between the fiber volume fraction and the fiber area fraction. The results were compared to Christensen [6], Manera [8] results and data reported in literature as well. The results agreed with the experimental data at low fiber volume fractions. Huang [10] presented an approach to obtain the effective moduli of 2D and 3D randomly oriented short fibers composites in terms of shape and volume fraction of fibers. The fibers were treated as spheriodal inclusions to enable their geometry ranging from short to continuous fibers. A probability density function was used along with Mori-Tanaka mean field theory to consider the interaction between fiber and matrix. Numerical examples showed that the effective moduli are strongly affected by fiber volume fraction, fiber aspect ratio and fiber orientation. Jiang et al [11] discussed the effect of aspect ratio and fiber orientation on the overall elastic properties of short fiber reinforced composites using the Mori-Tanaka method. They found differences more than 30% in the overall properties obtained from the model using average aspect ratio when the aspect ratio did not follow a symmetric distribution. Liang [12] derived an expression for Young's modulus of short inorganic fiber reinforced polymer composites based on previously proposed work of the same author [13]. The equation was verified by comparing its results to experimental data reported in literature showing good agreement.

The quick review above shows the importance of modeling the elastic properties of randomly oriented short fiber reinforcing composites. Although empirical modeling shows success in this area, it requires time and effort to prepare and test samples. An analytical model that gives reasonable and comparable results can be beneficial and efficient in understanding the influence of various parameters on the elastic properties of composites.

In this work, analytical model utilizes Mori-Tanaka modified approach [15] is merged with a randomization procedure to calculate the elastic properties of randomly oriented short fiber reinforced polymer composites. Furthermore, the effect of fiber volume fraction and fiber's aspect ratio (fiber length/fiber diameter) on the elastic properties of composite material will be discussed.

2. Micromechanics model

The modified approach of Mori-Tanaka that utilizes Eshelby tensors [14] will be used. Perfect bonding between fiber and surrounding matrix is assumed. Applying displacement boundary conditions that cause a uniform overall strain (ϵ_o), the average stress ($\bar{\sigma}$) on a volume element can be defined as [15]:

$$\bar{\sigma} = C_c \cdot \bar{\epsilon} = C_c \cdot \epsilon_o \text{ .... (1)}$$

Where, C_c is the tensor of elastic moduli of composite and $\bar{\epsilon}$ is the average strain on the volume element. Considering the volume fraction (V_j) of each phase (j), the following general expressions of the volume average of the stress and strain can be obtained [16].

$$\bar{\sigma} = V_f \cdot \langle\bar{\sigma_f}\rangle + V_m \cdot \overline{\sigma_m}$$

$$\bar{\epsilon} = V_f \cdot \langle\bar{\epsilon_f}\rangle + V_m \cdot \overline{\epsilon_m} \text{ .... (2)}$$

Where: the subscript ***f*** and ***m*** represents the phases of fiber and matrix respectively and the angle brackets denotes the average strain of randomly oriented fibers. Using equations 1 and 2, the effective tensor of elastic moduli of the composite can be obtained as:

$$C_c = V_f \cdot C_f \cdot A_f + V_m \cdot C_m \cdot A_m \cdots (3)$$

Where, A_j are fourth order tensors representing the strain concentration factors that relate the average strain in each phase to the average strain in composite as follows:

$$\bar{\epsilon_j} = A_j \cdot \bar{\epsilon} = A_j \cdot \epsilon_o \text{ , and}$$

$$\textstyle\sum_j V_j \cdot A_j = I \text{ .... (4)}$$

Where, ***I*** is the fourth order identity tensor. Replacing the stress concentration factors by the dilute strain concentration tensor following [16], the effective tensor for the elastic moduli can be obtained from:

$$C_c = C_m + \left[V_f\langle(C_f - C_m)A_f^d\rangle\right] \times$$

$$\left[V_m I + V_f\langle A_f^d\rangle\right]^{-1} \text{ .... (5)}$$

Where, A_f^d is the local dilute strain concentration fourth order tensor given by:

$$A_f^d = \left[I + S(C_m)^{-1}(C_f - C_m)\right]^{-1} \text{ ...... (6)}$$

Where, ***S*** is the Eshelby fourth order tensor [14].

2.1 Randomization procedure

For randomly distributed fibers in 3D, the relationship between the local axis system of the fiber and its global axis system can be defined using the transformation tensor (*T*). Figure 1 shows the local axis system for the short fiber (x, y and z) and the composite's global axis system (X, Y and Z). Two Euler angles (α, β) is used in the transformation matrix (*T*) to perform the needed transformation of the moduli tensor.

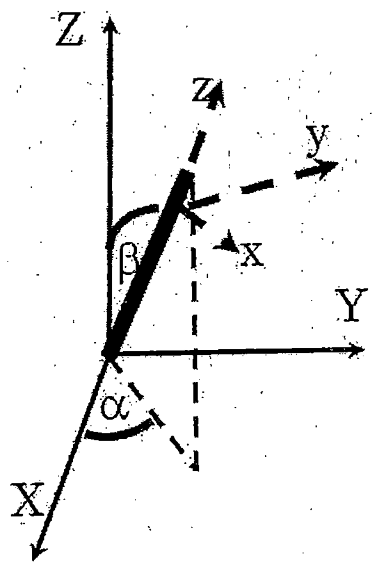

Figure 1: Euler angles defining the relationship between the local and global axis systems.

The fourth order local dilute strain tensor A^d_{pqrs} can be transformed into the global A^d_{ijkl} using the transformation tensor T as follow [17]:

$$A^d_{ijkl} = T_{ip}T_{jq}T_{kr}T_{ls}A^d_{pqrs} \quad \text{............} (7)$$

And the average of the dilute mechanical concentration fourth order tensor for random orientation can be calculated as follow [18, 19]:

$$\langle A \rangle = \frac{\int_0^{\pi}\int_0^{\pi} A^d(\alpha,\beta)\sin\beta \; d\alpha \, d\beta}{\int_0^{\pi}\int_0^{\pi} \sin\beta \, d\alpha \, d\beta} \quad \dots (8)$$

Where, the angle brackets indicate the average values over all orientations in the three dimensions (3D).

3. Model verification

The above equations in conjunction with randomization procedure are programmed in a MatLab® code to calculate the elastic properties of polymer composites. The model is able to calculate the composite elastic properties using elastic properties of composite constituents (fiber and matrix) at various fiber volume fractions and fiber's aspect ratios. The model results are compared to experimental data in literature [20] to verify model's accuracy. The experimental study used glass fibers dispersed randomly in polystyrene matrix to produce polymer composite materials with fiber volume fractions varies between 0.05 - 0.5. Elastic modulus of glass fiber is 72.395 GPa, Poisson's ratio is 0.2 and fiber diameter is 9.114 μm. Elastic modulus of polystyrene matrix is 3.24 GPa and Poisson's ratio is 0.32. Figure 2 shows the model results for elastic modulus of composite materials compared to the experimental data captured from literature [20]. The model results are calculated considering the randomization procedure (solid line) and also when considering the fibers are aligned in the direction of calculated modulus (dashed line).

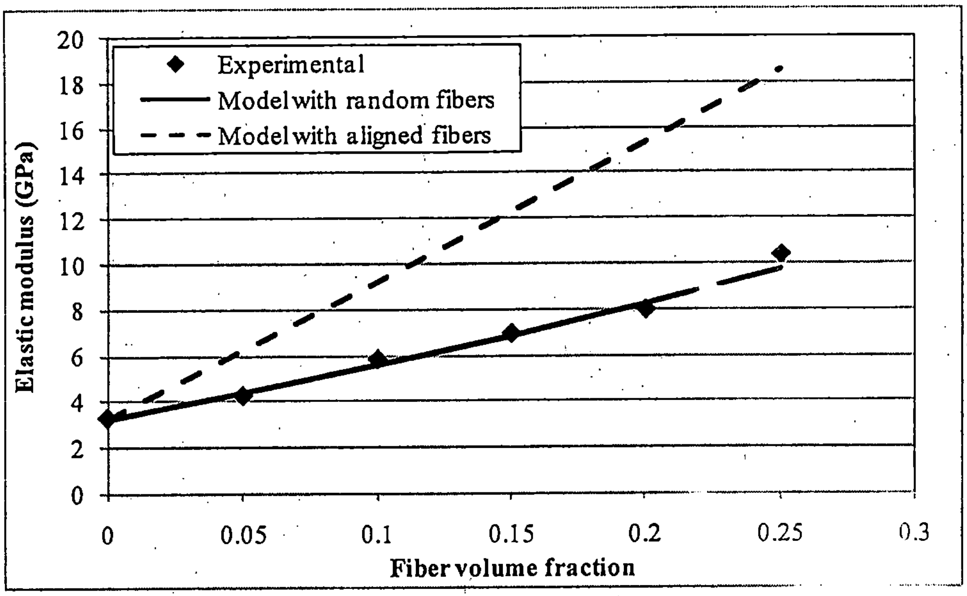

Figure 2: Comparison of elastic modulus from present model with/without random procedure and experimental data from [20].

Statistically, the model results, when the randomization procedure is considered, agree very well with the experimental data showing correlation coefficient equal to 0.992. On the other hand, the model results are overestimated when randomization procedure is ignored and the fibers are considered aligned. The overestimated results increase more significantly when fiber volume fraction increased. The correlation coefficient estimated in this case was 0.740.

4. Elastic properties of composites

A numerical case study is performed in this section to discuss the effect of fiber volume fraction and fiber's aspect ratio (α = fiber length/fiber diameter) on the elastic properties of randomly oriented short fiber reinforced polymer composite. The same material properties that is used in the previous section will be used here with fiber volume fractions varies from 0.05 to 0.7. Further, different aspect ratios will be used at every fiber volume fraction. Aspect ratios used are 10, 20, 30, 40. 50 and 100.

Figures 3-5 show the model results of the elastic properties of glass fibers reinforced polystyrene matrix composites considering the random orientations of fibers. Figures 3 and 4 show the effect of changing fiber volume fractions on both elastic and shear moduli at different fiber's aspect ratios.

Both moduli nonlinearly increase when the fiber volume fraction increases. A slight effect of changing fiber's aspect ratio shows up at higher fiber volume fractions. At higher fiber volume fraction, increasing fiber's aspect ratio over 30 almost has no effect on changing the composite's moduli.

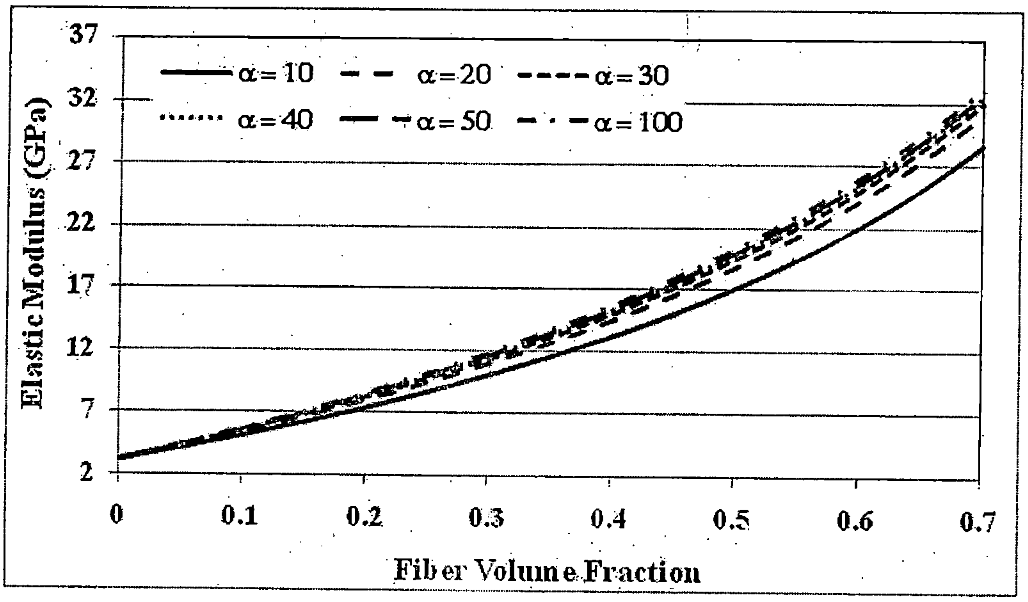

Figure 3: The effect of fiber volume fraction on the elastic modulus at different fiber's aspect ratios.

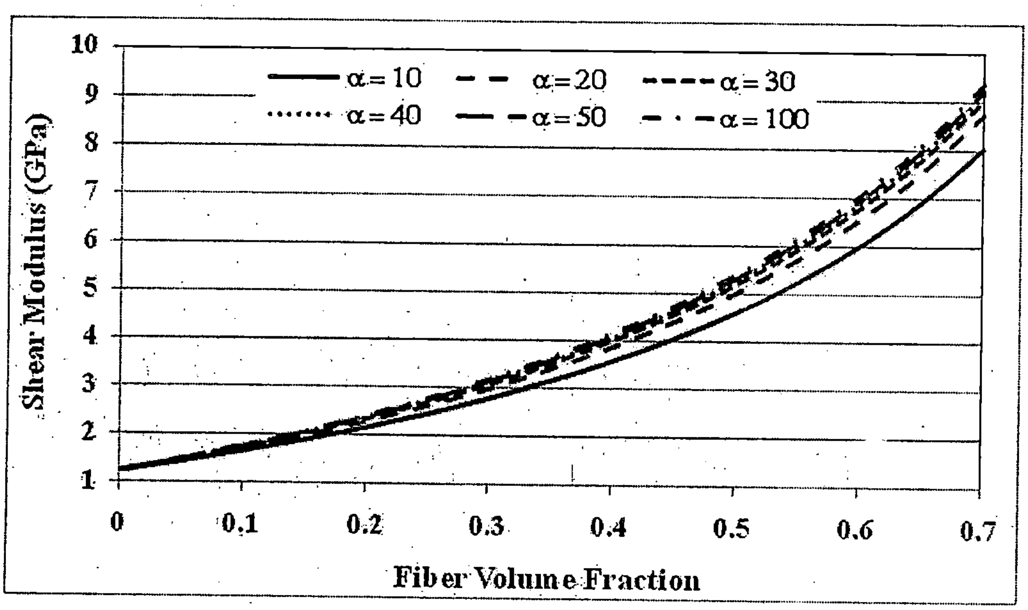

Figure 4: The effect of fiber volume fraction on the shear mod us at diffc.ent fiber's aspect ratios.

Figure 5 shows the effect of changing fiber volume fraction on the Poisson's ratio of the composite material at different fiber's aspect ratios. The Poisson's ratio of composite material nonlinearly decreases with increasing the fiber volume fraction. At low fiber volume fractions (up to 0.25) the effect of changing fiber's aspect ratio is not recognized. At higher volume fractions, small difference starts showing up. However, no recognized difference between results is shown when fiber's aspect ratio passes 30.

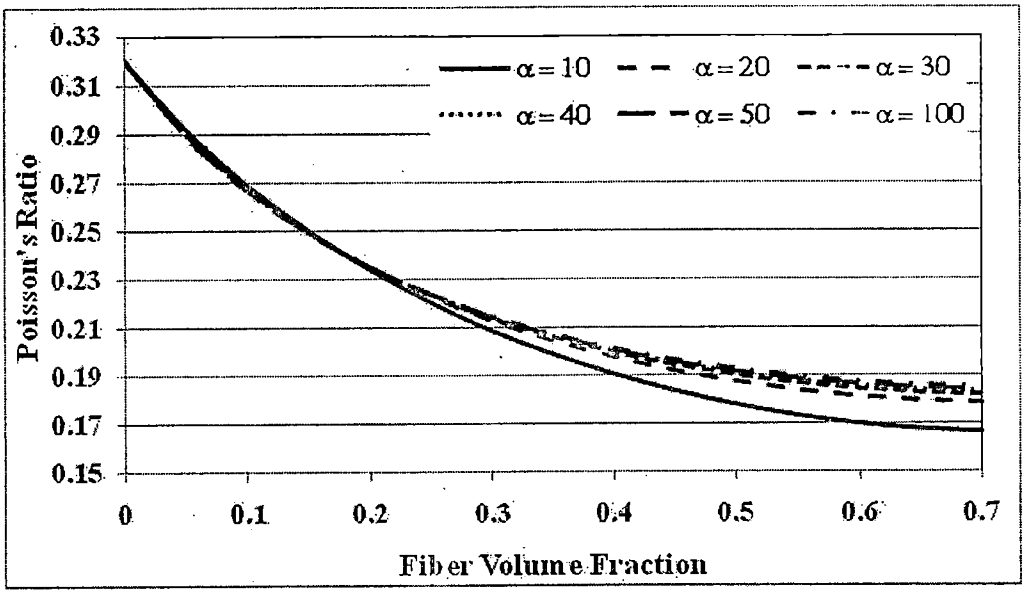

Figure 5: The effect of fiber volume fraction on the Poisson's ratio at different fiber's aspect ratios.

5. Conclusions

This study introduces analytical model that is based on the modified Mori-Tanaka approach along with randomization procedure to calculate the elastic properties of short fiber reinforced polymer composites. The model results were verified by comparing them to experimental data presented earlier in literature. The model results agree well with the experimental data only when the randomization procedure is induced. Thereafter, the model is used to discuss the effect of fiber volume fraction and fiber's aspect ratio on the elastic properties of polymer composites. The results show that the elastic and shear moduli are increased nonlinearly when the fiber volume fraction increases. Poisson's ratio of polymer composite is nonlinearly decreased when the fiber volume fraction increases. The study shows that the fiber volume fraction has a nonlinear effect on the composite elastic properties. On the other hand, increasing fiber's aspect ratio has no recognized effect on the composite elastic properties especially at low fiber volume fractions. At higher fiber volume fractions, very small variations show up when fiber's aspect ratio changed from 10 to 20. For fiber's aspect ratio equal or greater than 30, there is almost no effect on the composite elastic properties.

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